TECHNICAL REPORT ARLCB-TR-84025

WIDE RANGE DISPLACEMENT EXPRESSIONS FOR STANDARD FRACTURE MECHANICS SPECIMENS

J. A. KAPP

G. S. LEGER

BERNARD GROSS

JULY 1984

US ARMY ARMAMENT RESEARCH AND DEVELOPMENT CENTER
LARGE CALIBER WEAPON SYSTEMS LABORATORY
BENÉT WEAPONS LABORATORY
WATERVLIET N.Y. 12189

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER ARLCB-TR-84025	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (*and Subtitle*) WIDE RANGE DISPLACEMENT EXPRESSIONS FOR STANDARD FRACTURE MECHANICS SPECIMENS		5. TYPE OF REPORT & PERIOD COVERED Final 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) J. A. Kapp, G. S. Leger, and Bernard Gross (CONT'D ON REVERSE)		8. CONTRACT OR GRANT NUMBER(s)
9. PERFORMING ORGANIZATION NAME AND ADDRESS US Army Armament Research & Development Center Benet Weapons Laboratory, DRSMC-LCB-TL Watervliet, NY 12189		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS AMCMS No. 6111.02.H600.011 PRON No. 1A325B541A1A
11. CONTROLLING OFFICE NAME AND ADDRESS US Army Armament Research & Development Center Large Caliber Weapon Systems Laboratory Dover, NJ 07801		12. REPORT DATE July 1984 13. NUMBER OF PAGES 25
14. MONITORING AGENCY NAME & ADDRESS(*if different from Controlling Office*)		15. SECURITY CLASS. (*of this report*) UNCLASSIFIED 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (*of this Report*)

Approved for public release; distribution unlimited.

17. DISTRIBUTION STATEMENT (*of the abstract entered in Block 20, if different from Report*)

18. SUPPLEMENTARY NOTES

Presented at Sixteenth National Symposium on Fracture Mechanics,
Columbus, Ohio, 15-18 August 1983.
Published in proceedings of the symposium.

19. KEY WORDS (*Continue on reverse side if necessary and identify by block number*)

Fracture Mechanics
Testing Methods
Specimen Characterization

20. ABSTRACT (*Continue on reverse side if necessary and identify by block number*)

Wide range algebraic expressions for the displacement of cracked fracture mechanics specimens are developed. For each specimen two equations are given: one for the displacement as a function of crack length and the other for crack length as a function of displacement. All of the specimens that appear in ASTM Standard E-399 are represented in addition to the crack mouth displacement for a pure bending specimen. For the compact tension sample and the disk-shaped

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7. AUTHORS (CONT'D)

G. S. Leger
Graduate Student
Mechanical Department
University of New Mexico
Albuquerque, NM 87106

Bernard Gross
Materials Engineer
NASA-Lewis Research Center
Cleveland, OH 44135

20. ABSTRACT (CONT'D)

compact tension sample, the displacement at the crack mouth and at the load line are both considered. Only the crack mouth displacements for the arc-shaped tension samples are presented. The agreement between the displacements or crack lengths predicted by the various equations and the corresponding numerical data from which they were developed are nominally about three percent or better. These expressions should be useful in all types of fracture testing including J_{Ic}, K_R, and fatigue crack growth.

TABLE OF CONTENTS

TABLES

LIST OF ILLUSTRATIONS

INTRODUCTION

In order to determine the many fracture properties available to characterize materials, several parameters of the specimens used must be known. Foremost among these parameters is the crack length. Measuring properties such as J-resistance curves, K-resistance curves, and fatigue crack growth rates, the crack length changes must be measured during the actual test. Several techniques have been devised to measure crack growth by instrumenting the sample. Perhaps the simplest method is the so-called "compliance" technique where the elastic compliance of the specimen is measured by simultaneously measuring the load and displacement of the sample. Since the elastic load-displacement characteristics of any cracked body are a function of the elastic properties of the material tested and specimen parameters including crack length, the elastic properties or the crack length for any sample can be determined if the other is known.

Presented in this report are algebraic equations that allow for the easy calculation of either crack length or elastic properties for many standardized specimens.

In order to determine the elastic properties of the material from which the specimen was made, an expression which gives the load-displacement characteristics as a function of crack length must be developed. Similarly, to calculate the crack length, an expression must be developed where the crack length is a function of the load and displacement measured. Expressions of both types were developed for all the standardized specimens in ASTM E-399 and for a rectangular pure bending sample.

The expressions are developed by first establishing the appropriate limiting displacements as the crack approaches zero length and as the remaining ligament approaches zero. This is accomplished using the Paris equation (ref 1) based on Castigliano's theorem. These limiting solutions serve as guides when choosing a nondimensional form of the specimen displacements which has finite limits at short and long crack lengths. Numerically determined displacements are then normalized to the nondimensional form derived from the limiting solutions, and multivariable regression is used to fit a polynomial to these data. The resulting algebraic equations represent the displacement solutions nominally within three percent over a wide range of specimen parameters.

PROCEDURE TO DEVELOP NONDIMENSIONAL DISPLACEMENTS

To determine an appropriate nondimensional form of displacements for the various specimens considered, Paris's application of Castigliano's theorem to crack problems (ref 1) was used. For the general two-dimensional cracked body shown in Figure 1, the displacement due at the location at the applied force F in the direction of F is:

$$\delta_F = \frac{2}{E'} \int_{a_F}^{a} K_p \frac{\partial K_F}{\partial F} \, da \qquad (1)$$

where K_p is the stress intensity factor due to the force P, K_F is the stress intensity factor due to the force F, and E' is Young's Modulus (E) for plane stress or $E/(1-\nu^2)$ (ν = Poisson's ratio) for plane strain.

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[1]Tada, H., Paris, P., and Irwin, G., The Stress Analysis of Cracks Handbook, Del Research Corp., Hellertown, PA, 1973.

For the short crack limit we use the K solution for a finite crack in a semi-infinite medium. By placing a dummy load F at the crack mouth and replacing the load P with a uniform stress σ, the crack mouth opening displacement can be derived. The various K solutions necessary for Eq. (1) are (ref 2):

$$K_p = 1.12\sigma\sqrt{\pi a}$$

$$K_F = \frac{2.60F}{\sqrt{\pi a}}$$

$$\frac{\partial K_F}{\partial F} = \frac{2.60}{\sqrt{\pi a}}$$

Substituting into Eq. (1) we obtain:

$$\delta_{crack\ mouth} = \frac{5.824\sigma a}{E'} \tag{2}$$

A similar approach can be taken for the long crack limit. In this case, we assume that for tension samples, the normal component is negligible. Thus, only one K solution is necessary, namely that of a semi-infinite crack in a semi-infinite medium subjected to a moment M. For Eq. (1), $K_F = K_p = 3.975M/b^{3/2}$, where b is the uncracked ligament (W-a). With this approach integrating Eq. (1) gives us the angle of rotation of the two crack surfaces θ_m

$$\theta_m = \frac{15.8M}{E'(W-a)^2B}$$

[2] Saxerra, A. and Hudak, S. J., Jr., International Journal of Fracture, Vol. 14, No. 5, October 1978, pp. 453-468.

where B is the out-of-plane thickness of the specimen. The crack mouth displacement is estimated as the arc length swept out by θ_m at the distance W, or

$$\delta_{crack\ mouth} = \frac{15.8\ MW}{E'(W-a)^2 B} \qquad (3)$$

Using Eqs. (1) and (2) we can derive the appropriate nondimensional form of displacement based on the loading conditions and specimen parameters of the individual samples being considered. This enables us to fit an expression to the available data that gives the displacement as a function of crack length. Manipulating these equations will result in a mathematical form of displacements which is a function of crack length alone and has finite limits as the normalized crack length (a/W) approaches both zero and one.

Developing an expression to calculate the crack length as a function of measured displacements is not as straightforward as the above procedure. A first approach would be to solve for crack length in either Eq. (2) or Eq. (3) and use this as a nondimensional form of displacement. For example, Eq. (3) can be written as:

$$\frac{a}{W} = 1 - \left(\frac{15.8M}{E'BW\delta}\right)^{1/2} \qquad (4)$$

which applies as $\delta \to \infty$, but when δ goes to zero, Eq. (4) predicts a crack length of negative infinity when it should give a value of zero. Equation (4) can serve as guidance in selecting a form of nondimensional displacement with reasonable limits that can be used to accurately calculate crack length. We assume that:

$$\frac{a}{W} = f(\delta') \qquad (5)$$

$$\delta' = \frac{1}{1 + \left(\frac{15.8M}{E'BW\delta}\right)^{1/2}} \tag{6}$$

It is clear that as the displacement goes to zero, δ' also goes to zero and as δ goes to infinity, δ' goes to one. These limits enable the function $f(\delta')$ to be fit to the numerical data with great accuracy. A form of displacement similar to Eq. (6), has been suggested (ref 2) for a compact specimen.

Some variation of Eqs. (2), (3), and (6) is necessary when finding expressions for the load line displacements of the three-point bend sample and for the arc tension sample. But the resulting nondimensional form of displacements is only subtly different from the above.

RESULTS AND DISCUSSION

All of the specimens studied are shown in Figure 2. Three cases for the bending sample are presented: crack mouth opening displacement under pure bending, crack mouth opening displacement, and load line deflection under three-point bending. The three tension samples considered were the compact tension, the disk-shaped compact tension, and the arc-shaped tension samples. Expressions are presented for the load line displacement and the crack mouth opening displacement for all the tension samples except the arc-shaped sample. Only crack mouth opening was considered for this sample.

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[2]Saxerra, A. and Hudak, S. J., Jr., International Journal of Fracture, Vol. 14, No. 5, October 1978, pp. 453-468.

Bending Samples

For the crack mouth opening displacement under pure bending, the short and long crack limits using Eqs. (2) and (3) are:

$$\lim_{a/W \to 0} \frac{E'BW\delta}{15.8M(a/W)} = 2.212 \tag{7}$$

$$\lim_{a/W \to 1} \frac{E'BW\delta(1-a/W)^2}{15.8M} = 1 \tag{8}$$

Thus, we can represent the variation in displacements with (a/W) as

$$\frac{E'BW\delta(1-a/W)^2}{15.8M(a/W)} = f_{PBCM}(a/W) \tag{9}$$

Normalizing the numerical data for this sample (ref 3) according to Eq. (9), f_{PBCM} was found to be:

$$f_{PBCM}(a/W) = 2.212 - 4.78\ a/W + 7.37\ (a/W)^2 + 0.0830\ (a/W)^3 - 10.4\ (a/W)^4 + 6.53\ (a/W)^5 \tag{10}$$

Representing a/W as a function of displacements, we used the following:

$$\delta' = \frac{1}{1 + \left(\frac{15.8M}{E'BW\delta}\right)^{1/2}} \tag{11}$$

$$a/W = f'_{PBCM}(\delta') \tag{12}$$

Equation (12) has the limits of zero as a/W approaches zero and one as a/W goes to one. Again, normalizing the numerical data f'_{PBCM} can be developed as

$$f'_{PBCM}(\delta') = -0.98\delta' + 5.150\delta'^2 - 4.28\delta'^3 1.11\delta'^4 \tag{13}$$

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[3]Gross, B., Roberts, E., Jr., and Srawley, J. E., International Journal of Fracture Mechanics, Vol. 4, 1968, p. 267.

The accuracy of Eqs. (10) and (13) is demonstrated by the comparison shown in Table I. In Eq. (10) the displacement can be represented within ± 1.5% for all a/W ratios, and the crack length can also be predicted within ± 0.7% W for any displacement from Eq. (13).

For the crack mouth opening displacement of the three-point bend sample, the short and long crack limits are essentially the same as Eqs. (7) and (8). The value of the moment is replaced by PS/4, thus these limits are

$$\lim_{a/W \to 0} \frac{E'B\delta}{3.95P(S/W)(a/W)} = 2.212 \tag{14}$$

$$\lim_{a/W \to 0} \frac{E'B\delta(1-a/W)^2}{3.95P(S/W)} = 1 \tag{15}$$

The nondimensional form of displacement for fitting is then

$$\frac{E'B\delta(1-a/W)^2}{3.95P(S/W)(a/W)} = f_{3PBCM}(a/W) \tag{16}$$

$$f_{3PBCM}(a/W) = 2.21 - 6.57\ a/W + 17.9(a/W)^2 - 26.6(a/W)^3 + 19.9(a/W)^4 - 5.86(a/W)^5 \tag{17}$$

To predict the crack length from displacement measurements, the same form as above was utilized:

$$\delta' = \frac{1}{1 + \left(\frac{3.95P(S/W)}{E'B\delta}\right)^{1/2}} \tag{18}$$

$$a/W = f'_{3PBCM}(\delta')$$

$$f_{3PBCM}(\delta') = -1.03\delta' + 6.00\delta'^2 - 6.37\delta'^3 + 2.73\delta'^4 - 0.321\delta'^5 \tag{19}$$

Comparing Eqs. (17) and (19) with the numerical data (ref 3) in Table II shows again that the regression equations fit the data within ± 3.4%.

The load line deflection of the three-point bend sample requires more algebra than the two cases already presented. With no crack present, the beam will deflect due to the applied load. Thus the total deflection is the deflection with no crack plus the additional deflection caused by the introduction of the crack. The limiting solutions for the total deflection δ_{tot} then are:

$$\lim_{a/W \to 0} \delta_{tot} = \frac{P(S/W)^3}{4E'B} + \frac{8.891P(S/W)^2(a/W)^2}{E'B} \tag{20}$$

$$\lim_{a/W \to 1} \delta_{tot} = \frac{P(S/W)^3}{4E'B} + \frac{0.9875P(S/W)}{E'B(1-a/W)^2} \tag{21}$$

Fitting the numerical data we used the following form of nondimensional deflection:

$$\frac{EB\delta_{tot}}{P(S/W)^2} = \frac{(S/W)}{4} + \frac{(a/W)^2}{(1-a/W)^2} f_{3PBLL}(a/W) \tag{22}$$

$$f_{3PBLL}(a/W) = 8.89 - 33.9\ a/W + 68.5(a/W)^2 - 68.1(a/W)^3 + 25.6(a/W)^4 \tag{23}$$

Of the total displacement, the form used was

$$\delta' = \frac{1}{1 + \left(\dfrac{.9875}{\dfrac{E'B\delta_{tot}}{P(S/W)^2} - \dfrac{S/W}{4}}\right)^{1/2}} \tag{24}$$

$$(a/W) = f'_{3PBLL}(\delta') \tag{25}$$

$$f'_{3PBLL}(\delta') = 0.0997 - 0.516\delta' + 2.85\delta'^2 - 1.43\delta'^3 \tag{26}$$

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[3]Gross, B., Roberts, E., Jr., and Srawley, J. E., International Journal of Fracture Mechanics, Vol. 4, 1968, p. 267.

Table III compares the numerically determined load line displacements (ref 1) with Eqs. (23) and (26) for the case when $S/W = 4$. As the table indicates, the displacement as a function of a/W is accurate within $\pm$ 1.7% for any a/W and the crack length can be predicted from displacements within about $\pm$ 0.4% for $a/W > 0.2$.

Pin-Loaded Specimens

Fitting expressions for the displacement of pin-loaded specimens is somewhat more difficult than with bending samples. We are unable to use the short crack limit because that requires knowledge of the short crack K solution. In the compact tension and disk-shaped compact tension configurations, interactions with the pin loading holes must be considered when the relative crack length is less than about 0.2. Although the loading holes are not a factor with the arc-shaped tension, the value of the radius ratio (r_2/r_1) strongly affects K for that sample at short crack depths. We will concern ourselves only with the displacement characteristics for tension samples when a/W is greater than 0.2. Thus, only the deep crack limit is considered. For both the compact tension and the disk-shaped compact tension, Eq. (3) becomes

$$\lim_{a/W \to 1} \delta = \frac{15.8P}{E'B(1-a/W)^2} \tag{27}$$

This suggests that an appropriate nondimensional form of displacement for the compact specimen is

$$\frac{E'B\delta(1-a/W)^2}{15.8P} = f(a/W) \tag{28}$$

It should be noted that Eq. (27) applies only for load line displacements. Remember that Eq. (3) was derived from the rotation of the crack

[1] Tada, H., Paris, P., and Irwin, G., The Stress Analysis of Cracks Handbook, Del Research Corp., Hellertown, PA, 1973.

surfaces θ_M, and the displacement was estimated as the arc length at a distance W from the uncracked ligament. At the crack mouth, the displacement would be the arc length approximated by $\theta_M \times (W+\Delta)$, where Δ is the distance from the load line to the crack mouth. Using this formulation would result in a rather messy equation. For this reason we chose to use Eq. (28) as the non-dimensional form of displacements and restrict the applicability of the resulting expression to values of (a/W) less than 0.8.

Four polynomials were found using Eq. (28): load line displacements for both specimen types and crack mouth displacements for both specimens. They are:

Compact Tension - Load Line:

$$f_{CTLL}(a/W) = 0.121 + 1.21(a/W) - 0.159(a/W)^2 - 1.47(a/W)^3 + 1.30(a/W)^4 \quad (29)$$

Disk-Shaped Tension - Load Line:

$$f_{DTLL}(a/W) = 0.104 + 1.11(a/W) - 0.262(a/W)^2 + 0.0247(a/W)^3 + 0.0223(a/W)^4 \quad (30)$$

Compact Tension - Crack Mouth:

$$f_{CTCM}(a/W) = 0.631 + 0.178(a/W) + 1.96(a/W)^2 - 3.99(a/W)^3 + 2.48(a/W)^4 \quad (31)$$

Disk-Shaped Tension - Crack Mouth:

$$f_{DTCM} = 0.595 - 0.168(a/W) + 2.86(a/W)^2 - 3.10(a/W)^3 + 1.26(a/W)^4 \quad (32)$$

Comparisons between the numerical data for compact tension samples (refs 3,4) and Eqs. (29) and (31), and the data for disk-shaped compact tension

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[3]Gross, B., Roberts, E., Jr., and Srawley, J. E., International Journal of Fracture Mechanics, Vol. 4, 1968, p. 267.

[4]Newman, J. C., Jr., "Stress-Intensity Factors and Crack-Opening Displacements For Round Compact Specimens," NASA TM 80174, National Aeronautics and Space Administration, Langley Research Center, Hampton, VA, 1979.

samples (ref 4) and Eqs. (30) and (32) are shown in Table IV. The following accuracy statements can be made based on these comparisons. For compact tension load line displacements ±0.2% for $0.2 \leq a/W \leq 0.8$; for compact tension crack mouth displacements ± 0.2% for $0.2 \leq a/W \leq 0.8$; for disk-shaped compact tension load line displacements ± 0.3% for $0.2 \leq a/W \leq 0.8$; and for disk-shaped compact tension crack mouth displacements ± 0.2% for $0.2 \leq a/W \leq 0.8$.

To determine crack length as a function of displacements, we used the following variation of Eq. (6):

$$\delta' = \frac{1}{1 + \left(\frac{15.8P}{EB\delta}\right)^{1/2}} \tag{33}$$

$$(a/W) = f(\delta') \tag{34}$$

Again, four polynomials were generated for the two samples each with two displacement measuring locations. These equations are:

Compact Tension-Load Line:

$$a/W = -0.228 - 0.252\delta' + 4.60\delta'^2 - 4.41\delta'^3 + 1.29\delta'^4 \tag{35}$$

Disk-Shaped Tension-Load Line:

$$a/W = 0.0896 - 1.81\delta' + 7.61\delta'^2 - 7.19\delta'^3 + 2.30\delta'^4 \tag{36}$$

Compact Tension-Crack Mouth:

$$a/W = -1.052 + 1.38\delta' + 4.71\delta'^2 - 6.41\delta'^3 + 2.36\delta'^4 \tag{37}$$

Disk-Shaped Tension-Crack Mouth:

$$a/W = -1.292 + 3.55\delta' - 0.589\delta'^2 - 1.64\delta'^3 + 0.993\delta'^4 \tag{38}$$

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[4]Newman, J. C., Jr., "Stress-Intensity Factors and Crack-Opening Displacements For Round Compact Specimens," NASA TM 80174, National Aeronautics and Space Administration, Langley Research Center, Hampton, VA, 1979.

Table V gives the comparison between the numerical data for these two specimens (refs 3,4) and the a/W values predicted by Eqs. (35) through (38). From the table, it can be concluded that the statistical data fits have the following accuracies: compact tension-load line ± 0.3% for $a/W \geq 0.2$; disk-shaped tension-load line ± 0.1% for $a/W \geq 0.2$; compact tension-crack mouth ± 0.1% for $0.2 \leq a/W \leq 1.0$; disk-shaped tension-crack mouth ± 0.2% for $0.2 \leq a/W \leq 1.0$.

Arc-Shaped Tension Specimens

Because of the many different geometries available for the arc-shaped samples, the wide range expressions are somewhat more complicated. Again, we are not able to use the short crack limit but not because of loading hole interactions, rather because of the curvature at the inner radius. The stress at the inside radius is necessary to apply Eq. (2), which requires the curved beam theory. The resulting nondimensional form of displacement is very complex. Thus, the effects of the eccentric loading (the X/W dependence) and the effects of radius ratio, r_1/r_2 must also be accounted for. The long crack limit, Eq. (3) can be used and in terms of arc-shaped parameters we have:

$$\lim_{a/W \to 1} \frac{E'B\delta(1-a/W)^2}{(2X/W+1+a/W)} = 7.9 \tag{39}$$

This nondimensional form of displacement accounts for the X/W dependence for crack mouth displacements very nicely, but we still needed to account for

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[3]Gross, B., Roberts, E., Jr., and Srawley, J. E., International Journal of Fracture Mechanics, Vol. 4, 1968, p. 267.

[4]Newman, J. C., Jr., "Stress-Intensity Factors and Crack-Opening Displacements For Round Compact Specimens," NASA TM 80174, National Aeronautics and Space Administration, Langley Research Center, Hampton, VA, 1979.

the r_1/r_2 effects by some other means. Without the short crack limit, empirical techniques were necessary. Using Eq. (39) to normalize the available data generated for this report revealed that corrections for the r_1/r_2 effect of no more than about ten percent were necessary. These corrections are expressed in the following manner:

$$\frac{E'B\delta}{P} = \frac{2X/W+1+a/W}{(1-a/W)^2} f_{ATCM}(a/W, r_1/r_2)$$

$$f_{ATCM}(a/W,r_1/r_2) = 0.34 + 13.75\ a/W - 12.67(a/W)^2 + 6.47(a/W)^3 + $$
$$+ (1-a/W)^{0.05}(1-r_1/r_2)(0.8-0.5\ r_1/r_2) \qquad (40)$$

A comparison of the numerically generated displacements and Eq. (40) is given in Table IV. The accuracy of the above expression can be stated as: for $X/W = 0$ and $0.4 \leqslant r_1/r_2 \leqslant 0.91$, $\pm$ 2.9% for $0.2 \leqslant a/W \leqslant 0.8$, and $\pm$ 0.4% for $0.4 \leqslant a/W \leqslant 0.6$; for $X/W = 0.5$ and $0.4 \leqslant r_1/r_2 \leqslant 0.91$, $\pm$ 3.6% for $0.2 \leqslant a/W \leqslant 0.8$, and $\pm$ 1.5% for $0.4 \leqslant a/W \leqslant 0.6$.

For crack length as a function of displacement, we use a modification of Eq. (6):

$$\delta' = \frac{1}{1 + \left[\frac{7.9(2X/W + 1)P}{E'B\delta}\right]^{1/2}} \qquad (41)$$

This representation does not account for all of the X/W dependence or any of the r_1/r_2 effects. Again resorting to empirical methods, we can account for these effects by modifying Eq. (41) as:

$$\delta' = \frac{1}{1 + \left[\frac{7.9(2X/W + 1)P}{E'B\delta}\right]^{1/2}} + 0.4\ X/W + 0.016 + 0.017\ r_1/r_2 \qquad (42)$$

Using this form, the crack length can be expressed by:

$$a/W = -0.941\delta' + 4.253\delta'^2 - 3.460\delta'^3 + 1.146\delta'^4 \tag{43}$$

where δ' is calculated by Eq. (42).

Table VII compares Eq. (43) with the expected values of crack length. The errors as a percentage of W can be expressed as: for $X/W = 0$ and $0.4 \leq r_1/r_2 \leq 0.91$, $\pm$ 1.2% for $0.2 \leq a/W \leq 0.8$, and $\pm$ 0.6% for $0.4 \leq a/W \leq 0.6$; for $X/W = 0.5$ and $0.4 \leq r_1/r_2 \leq 0.9$, $\pm$ 1.9% for $0.2 \leq a/W \leq 0.8$ and $\pm$ 0.6% for $0.4 \leq a/W \leq 0.6$.

CONCLUSIONS

Wide range displacement expressions for many standard fracture testing specimens were generated by using limiting solutions to develop the proper nondimensional form. These expressions can be used to determine displacements when a/W is known or to determine crack length when the displacement and elastic properties of the specimen are known. These expressions may be useful when using the "compliance" method as a passive means of monitoring crack growth during fracture mechanics tests.

REFERENCES

1. Tada, H., Paris, P., and Irwin, G., The Stress Analysis of Cracks Handbook, Del Research Corp., Hellertown, PA, 1973.
2. Saxerra, A. and Hudak, S. J., Jr., International Journal of Fracture, Vol. 14, No. 5, October 1978, pp. 453-468.
3. Gross, B., Roberts, E., Jr., and Srawley, J. E., International Journal of Fracture Mechanics, Vol. 4, 1968, p. 267.
4. Newman, J. C., Jr., "Stress-Intensity Factors and Crack-Opening Displacements For Round Compact Specimens," NASA TM 80174, National Aeronautics and Space Administration, Langley Research Center, Hampton, VA, 1979.

TABLE I. CRACK MOUTH OPENING DISPLACEMENTS FOR PURE BENDING SAMPLE

a/W

	0	.2	.3	.4	.5	.6	.7	1.0
f_{PBCM} (ref 3)	2.212	1.530	1.390	1.281	1.213	1.193	1.097	1.0
f_{PBCM} (Eq. 10)	2.212	1.537	1.375	1.285	1.229	1.175	1.106	1.015
Error (%)	0	+0.5	-1.1	-0.3	1.3	-1.5	+0.8	1.5
δ'_{PBCM}	0	.4088	.4798	.5440	.6090	.6790	.745	1
a/W (Eq. 13)	0	.1995	.3026	.4006	.5010	.6073	.7032	1.005
Error (as % of W)	-	-0.1	+0.3	+0.1	+0.1	+0.7	+0.3	+0.5

TABLE II. CRACK MOUTH OPENING DISPLACEMENTS FOR THREE-POINT BEND SPECIMENS

a/W

	0	0.2	0.3	0.4	0.5	0.6	0.7	0.8	1.0
f_{3PBCM} (ref 3)	2.212	1.433	1.277	1.187	1.129	1.086	1.052	1.025	1
f_{3PBCM} (Eq. 17)	2.210	1.429	1.278	1.193	1.136	1.090	1.051	1.022	0.98
Error (%)	0.1	-0.3	0.5	+0.5	+0.6	+0.4	-0.1	-0.3	-2.0
δ' (ref 3)	0	0.4009	0.4693	0.5345	0.6004	0.6687	0.7737	0.8191	1
f'_{3PBCM} (Eq. 19)	0	0.2082	0.3048	0.3997	0.4955	0.5924	0.7338	0.7917	1.009
Error (as % W)	0	+0.8	+0.5	0	-0.5	-0.8	+3.4	-0.8	+0.9

TABLE III. LOAD LINE DEFLECTION OF THREE-POINT BEND SAMPLES

a/W

	0	.2	.3	.4	.5	.6	.7	.8	1.0
f_{3PBLL} (ref 1)	8.89	4.32	3.27	2.60	2.15	1.80	1.51	1.25	0.9875
f_{3PBLL} (Eq. 23)	8.89	4.35	3.25	2.59	2.15	1.82	1.51	1.23	0.99
Error	0	+0.6	-0.5	-0.5	0	+1.1	0	-1.7	+0.3
δ' (ref 1)	-	0.3434	0.4383	0.5197	0.5953	0.6695	0.7424	0.8179	1.000
f'_{3PBLL} (Eq. 26)	-	0.2007	0.3006	0.4006	0.5015	0.6026	0.7023	0.8018	1.004
Error (as % W)	-	+0.1	+0.1	+0.1	+0.2	+0.3	+0.2	+0.2	+0.4

TABLE IV. DISPLACEMENT FOR COMPACT TENSION SAMPLES AND DISK-SHAPED TENSION SAMPLES

a/W

	.2	.3	.4	.5	.6	.7	.8
f_{CTLL} (ref 4)	0.3465	0.4411	0.5195	0.5842	0.6408	0.6992	0.7671
f_{CTLL} (Eq. 29)	0.3470	0.4405	0.5188	0.5838	0.6407	0.6980	0.7671
Error (%)	+0.1	-0.1	-0.1	-0.1	0	-0.2	0
f_{DTLL} (ref 4)	0.3159	0.4154	0.5086	0.5984	0.6846	0.7672	0.8471
f_{DTLL} (Eq. 30)	0.3158	0.4134	0.5082	0.5980	0.6839	0.7664	0.8461
Error (%)	0	-0.3	-0.1	-0.1	-0.1	-0.1	-0.1
f_{CTCM} (ref 4)	0.7170	0.7722	0.8239	0.8661	0.9015	0.9426	1.000
f_{CTCM} (Eq. 31)	0.7170	0.7732	0.8239	0.8663	0.9030	0.9429	1.001
Error (%)	0	+0.1	0	0	+0.2	0	±0.1
f_{DTCM} (ref 4)	0.6459	0.7121	0.7906	0.8722	0.9521	1.029	1.103
f_{DTCM} (Eq. 32)	0.6458	0.7123	0.7905	0.8723	0.9527	1.030	1.105
Error (%)	0	0	0	0	+0.1	+0.1	+0.2

TABLE V. CRACK LENGTH AS A FUNCTION OF DISPLACEMENTS FOR COMPACT SAMPLES

(a/W)

	.2	.3	.4	.5	.6	.7	.8	1.0
δ'_{CTLL} (ref 4)	0.4239	0.4869	0.5457	0.6045	0.6668	0.7360	0.8141	1.0
a/W (Eq. 35)	0.1975	0.3033	0.4021	0.4987	0.5968	0.6986	0.8027	1.000
Error (% W)	-0.3	+0.3	+0.2	-0.1	-0.3	+0.1	+0.3	0
δ'_{DTLL} (ref 4)	0.4126	0.4793	0.5431	0.6074	0.6741	0.7449	0.8215	1.0
a/W (Eq. 36)	0.1999	0.3000	0.3995	0.4996	0.6000	0.7003	0.7998	0.9996
Error (% W)	0	0	-0.1	0	0	0	0	0
δ'_{CTCM} (ref 4)	0.5142	0.5566	0.6020	0.6505	0.7036	0.7639	0.8333	
a/W (Eq. 37)	0.2004	0.3005	0.4012	0.5009	0.6003	0.7009	0.8014	
Error (% W)	0	+0.1	+0.1	+0.1	0	+0.1	+0.1	
δ'_{DTCM} (ref 4)	0.5011	0.5466	0.5971	0.6513	0.7093	0.7717	0.8400	
a/W (Eq. 37)	0.1993	0.2973	0.3988	0.4999	0.5998	0.6993	0.8008	
Error (% W)	-0.1	-0.3	-0.1	0	0	-0.1	+0.1	

TABLE VI. CRACK MOUTH DISPLACEMENTS AS A FUNCTION OF CRACK LENGTH FOR ARC TENSION SAMPLES

$$\frac{E'B\delta}{P} = \frac{2X/W+1+a/W}{(1-a/W)^2} f(a/W, r_1/r_2)$$

$X/W = 0$

a/W

	0.2	0.3	0.4	0.5	0.6	0.7	0.8
r_1/r_2 = 0.9091							
$E'B\delta/P$	5.119	9.504	16.58	29.21	54.38	113.5	292.2
Eq. (40)	4.999	9.367	16.56	29.33	54.57	113.5	295.9
ERROR (%)	-2.3	-1.4	-0.1	+0.4	+0.4	0	+1.3
r_1/r_2 = .6667							
$E'B\delta/P$	5.356	9.843	17.07	29.98	55.67	115.8	302.6
Eq. (40)	5.323	9.696	17.04	30.07	55.81	115.8	301.4
ERROR (%)	-2.3	-1.5	-0.2	+0.3	+0.3	0	-0.4
r_1/r_2 = 0.5							
$E'B\delta/P$	5.621	10.17	17.54	30.68	56.77	117.7	304.9
Eq. (40)	5.456	10.01	17.51	30.78	57.00	118.0	306.7
ERROR (%)	-2.9	-1.5	-0.2	+0.3	+0.4	+0.3	+0.6
$X/W = 0.5$							
r_1/r_2 = 0.9091							
$E'B\delta/P$	8.848	16.24	27.97	48.52	88.80	181.8	458.6
Eq. (40)	9.165	16.57	28.39	48.87	88.69	180.2	460.3
ERROR (%)	+3.6	+2.0	+1.5	+0.7	-0.1	-0.9	+0.4
r_1/r_2 = .6667							
$E'B\delta/P$	9.300	16.91	28.39	50.00	91.16	185.9	475.3
Eq. (40)	9.592	17.16	29.21	50.11	90.69	183.9	468.9
ERROR (%)	+3.1	+1.5	+1.0	+0.2	-0.5	-1.0	-1.4
r_1/r_2 = 0.5							
$E'B\delta/P$	9.788	17.53	29.83	51.32	93.17	189.1	479.2
Eq. (40)	10.00	17.71	30.01	51.30	92.63	187.5	477.2
ERROR (%)	+2.2	+1.0	+0.6	-0.1	-0.6	-0.9	-0.4

TABLE VII. CRACK LENGTHS AS A FUNCTION OF CRACK MOUTH DISPLACEMENTS FOR ARC TENSION SAMPLES

$$\delta' = [1 + (\frac{P[2X/W+1]}{E'B\delta})^{0.5}]^{-1} + 0.4X/W + 0.016 + 0.017\ r_1/r_2$$

$$a/W = f(\delta')$$

$X/W = 0.0$

r_1/r_2 = 0.9091							
$E'B\delta/P$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Eq. (43)	.203	.304	.401	.498	.598	.703	.812
ERROR (%W)	+0.3	+0.4	+0.1	−0.2	−0.2	+0.3	+1.2
r_1/r_2 = .6667							
$E'B\delta/P$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Eq. (43)	.205	.305	.400	.496	.595	.699	.808
ERROR (%W)	+0.5	+0.5	0	−0.4	−0.5	−0.1	+0.8
r_1/r_2 = 0.5							
$E'B\delta/P$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Eq. (43)	.209	.306	.400	.594	.594	.696	.804
ERROR (%W)	+0.9	+0.6	0	−0.6	−0.6	−0.4	+0.4
X/W = 0.5							
r_1/r_2 = 0.9091							
$E'B\delta/P$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Eq. (43)	.206	.305	.400	.496	.597	.704	.819
ERROR (%W)	+0.6	+0.6	0	−0.4	−0.3	+0.4	+1.9
r_1/r_2 = .6667							
$E'B\delta/P$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Eq. (43)	.208	.306	.400	.495	.595	.701	.816
ERROR (%W)	+0.8	+0.6	0	−0.5	−0.5	+0.1	+1.6
r_1/r_2 = 0.5							
$E'B\delta/P$	0.2	0.3	0.4	0.5	0.6	0.7	0.8
Eq. (43)	.213	.308	.401	.495	.594	.698	.812
ERROR (%W)	+1.3	+0.8	+0.1	−0.5	−0.6	−0.2	+1.2

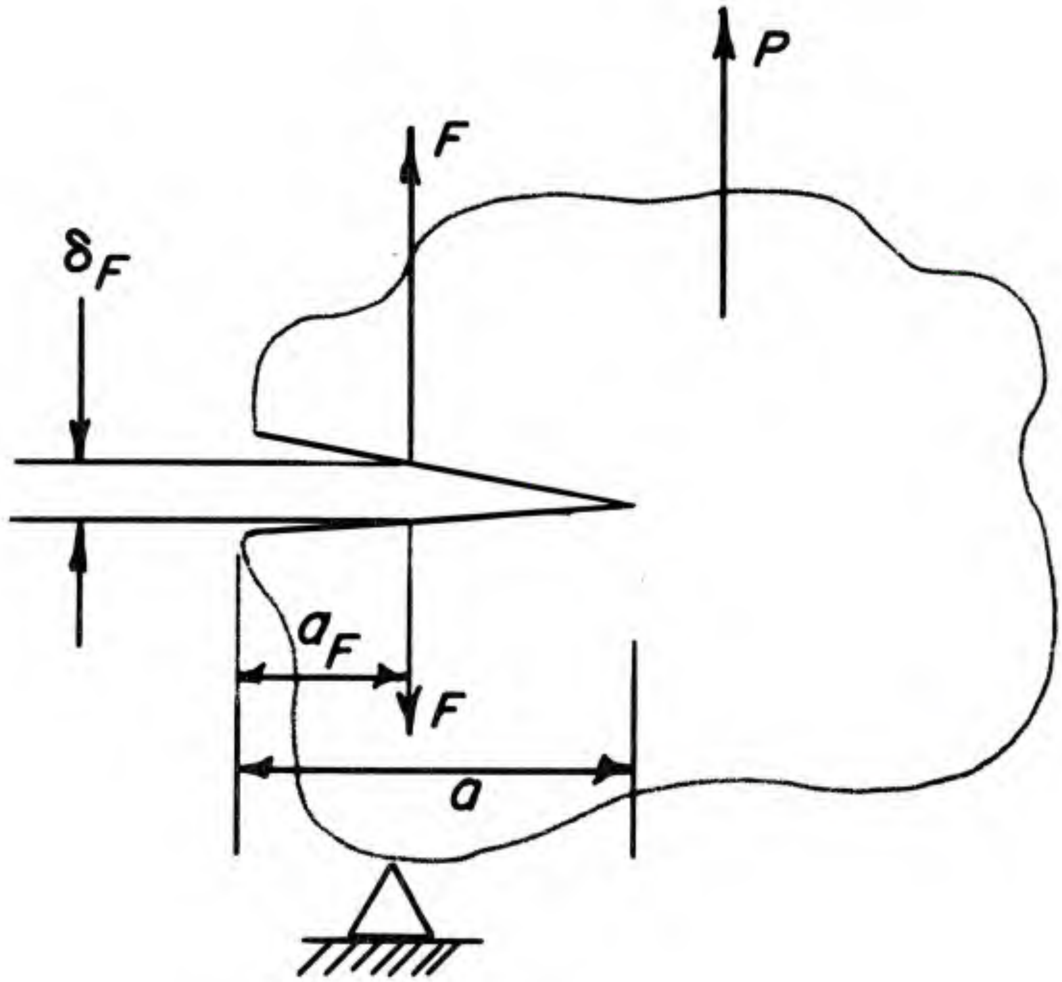

Figure 1. A general two-dimensional cracked body.

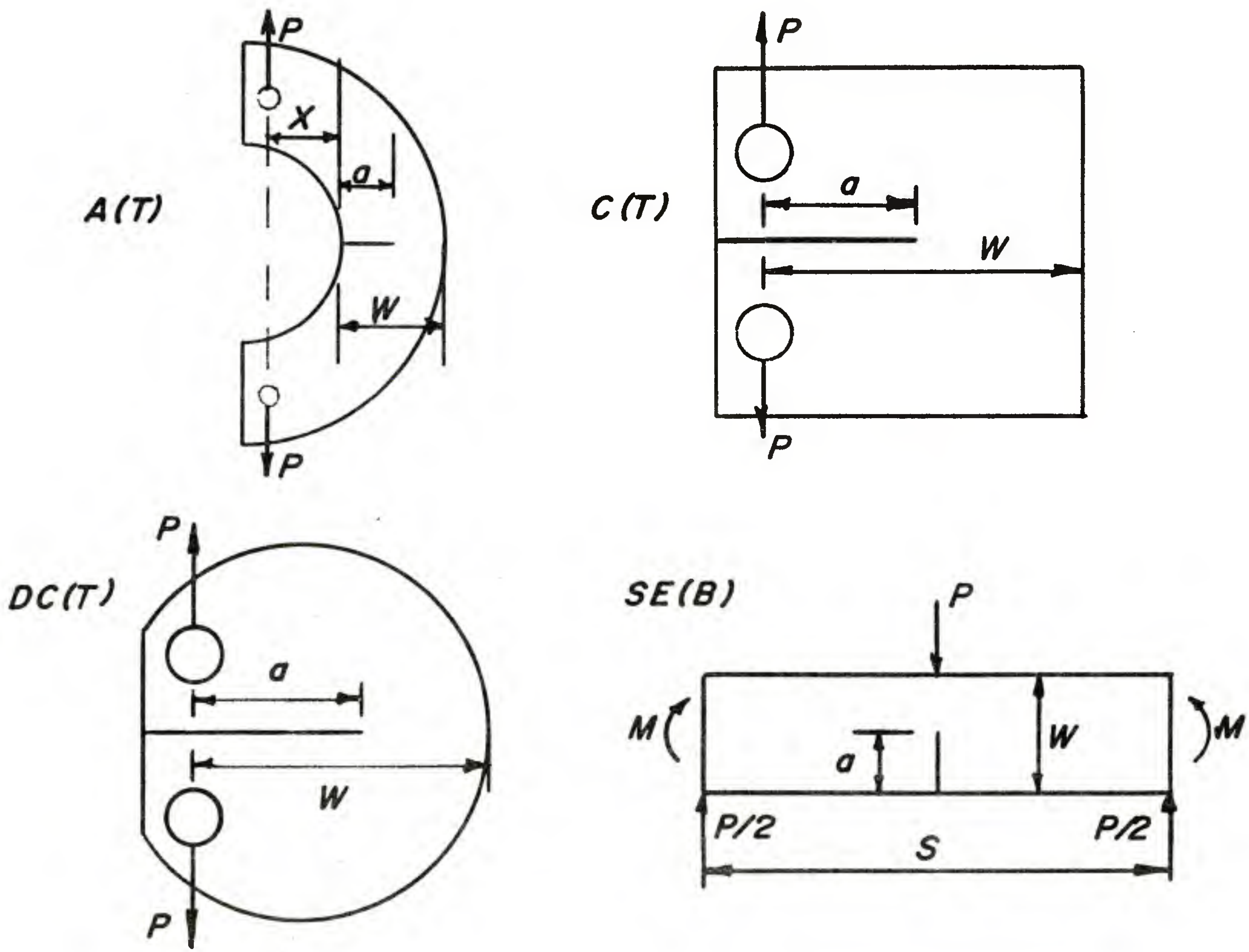

Figure 2. The specimens for which displacement expressions were generated.

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